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# GALAXY CLUSTERS FOR COSMOLOGY\*

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Using the gas mass fraction  $f_{\text{gas}}$  measurements obtained on the basis of X-ray data for two samples of hot and dynamically relaxed galaxy clusters: 42 clusters with redshifts in the range of  $0.05 < z < 1.1$  collected and analysed by Allen *et al.* (2008) and 35 clusters at redshifts  $0.15 < z < 0.30$  selected and analysed by Landry *et al.* (2013), we obtained constraints on main cosmological parameters in two popular cases:  $w$ CDM model in which dark energy equation of state is constant in time and the model in which dark energy equation of state evolves with redshift according to the Chevalier–Polarski–Linder (CPL) parametrization. Our results from numerical Monte Carlo calculations are following:  $\Omega_m = 0.3695^{+0.1121}_{-0.173}$ ,  $H_0 = 66.74^{+28.48}_{-42.15}$ ,  $w = -0.78^{+0.1695}_{-0.2615}$  for  $w$ CDM model and  $\Omega_m = 0.2523^{+0.1738}_{-0.0694}$ ,  $H_0 = 67.29^{+24.27}_{-29.98}$ ,  $w_0 = -0.86^{+0.4541}_{-0.3245}$ ,  $w_a = 0.6948^{+0.5226}_{-0.892}$  for CPL scenario and are in an agreement with the results based on other well-established, independent techniques. This shows that galaxy clusters can be used as a good tool in cosmology. Moreover, we investigate the recent (*i.e.* at low redshift) expansion history of the Universe finding no evidence that the cosmic acceleration is now slowing down.

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## 1. Introduction

The idea of using galaxy clusters as a tool for cosmology was firstly introduced with Zwicky’s discovery of the presence of dark matter in the Coma Cluster [1]. Since then, these objects have enjoyed unwavering popularity in the cosmological community [2]. The motivation is that they are the largest gravitationally bound objects in the Universe linking two regimes, at the smallest and at the largest scales, within cosmic structure formation process in an expanding Universe. According to the cosmology based

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on general relativity applied to homogeneous and isotropic metric, such an expansion depends strongly on the material components of the Universe. Within widely accepted in scientific community  $\Lambda$ CDM scenario revealing strong agreement with observational data, vast majority of matter is in an exotic form of dark energy responsible for an accelerating expansion of the Universe seen *e.g.* on the Hubble diagrams from supernovae surveys [3, 4] and pressureless, mainly non-baryonic (cold dark) matter visible *e.g.* in the flat rotation curves of galaxies or gravitational lensing data (see [5] and references therein).  $\Lambda$ CDM suffers, however, from several problems of fundamental nature [6] what motivates suggestions that it should be treated rather as a ‘concordance’ model. A number of alternative scenarios to  $\Lambda$ CDM have been proposed so far but, in fact, we still have no convincing signals (both on theoretical and observational side) of how to distinguish between them and find an answer for the question concerning a real nature of dark matter and dark energy puzzling phenomena. In the face of uncertainty, the only strategy which we have to our disposal is phenomenological approach relying on the estimation of values of cosmological parameters on the basis of data from different independent observations [7].

In this paper, we discuss in more details how gas mass fraction measurements for galaxy clusters may help to constrain values of relevant cosmological parameters, and thus, can be treated as a cosmological test alternative to other well-established methods. In particular, on the basis of the best-fitting values for cosmological parameters obtained by us through the analysis of the clusters data, we investigate recent expansion history of the Universe (*i.e.* deceleration parameter as a function of cluster redshift).

## 2. Motivation

There are several particular properties of galaxy clusters which allow to use them as a convincing tool in observational cosmology. Firstly, the matter content of such large structures in the Universe is considered as representative for the matter content in the Universe. In this light, we expect that ratio of baryonic to total mass in the clusters corresponds to the ratio of appropriate cosmological density parameters  $\Omega_b/\Omega_m$ . Secondly, the dominant baryonic mass fraction of galaxy clusters is in the form of hot gas (galaxies account for only about 2–3% of the total mass of the clusters) with temperature reflecting the gravitational potential and hence the total mass of the cluster. The ratio of the mass of this hot intracluster gas (ICG) to the total mass of the cluster — the gas mass fraction ( $f_{\text{gas}} = M_{\text{gas}}/M_{\text{tot}}$ ) — should, therefore, depend on  $\Omega_b/\Omega_m$ . On the other hand, one may deduce (see [8] for more details)  $f_{\text{gas}} \sim D_A^{1.5}$  dependence, where  $D_A$  is the angular diameter distance to the cluster. Because hydrodynamical simulations for

dynamically relaxed clusters [9] as well as the observations [8, 10] suggest that  $f_{\text{gas}}$  should not depend strongly on the redshift of the cluster, the gas mass fraction can serve also as a new kind of standard ruler for cosmology, helping in the cosmological parameters estimation process.

The main difficulty in the application of galaxy clusters in cosmology is the measurement of the ICG mass  $M_{\text{gas}}$  and the total mass  $M_{\text{tot}}$  of a given cluster. The ICG electrons have energies falling within the range of  $kT \sim 2\text{--}10$  keV what translates into the ICG temperature of  $T \sim (20\text{--}100) \times 10^6$  K. This allows them to radiate via bremsstrahlung and, in turn, such radiation can be visible by telescopes like Chandra or XMM Newton in the X-ray energy range. Thus, what we directly can observe for a given cluster located at some redshift  $z$  is the projected (*i.e.* integrated along the line-of-sight) X-ray surface brightness over a given frequency band [11, 12]

$$S_X = \frac{1}{4\pi(1+z)^4} \int n_e^2 \Lambda_{ee}(T_e, A) dl. \quad (1)$$

In the above formula,  $\Lambda_{ee}(T_e, A) \approx n_e^2 T_e^{-1/2} \exp(-E/k_B T_e)$  is the emissivity of ICG at a given energy  $E$  which depends on the ICG temperature  $T_e$  and metallicity  $A$  ( $k_B$  is a Boltzmann constant). The 3-dimensional electron number density  $n_e$  is usually assumed to follow the density profile of spherical isothermal  $\beta$  model [13, 14]

$$n_e(r) = n_{e0} \left( 1 + \frac{r^2}{r_c^2} \right)^{-3\beta/2}, \quad (2)$$

where  $n_{e0}$  is the central electron number density,  $r$  is the radius from the center of the cluster,  $r_c$  is the core radius of the intracluster gas, and  $\beta$  is a power law index. Thus, on the basis of measured X-ray surface brightness, one may obtain the temperature  $T_{\text{gas}}$  and density  $n_{\text{gas}}$  (pressure  $p_{\text{gas}}$ ) profiles of hot ICG for a given measurement radius beyond the core radius ( $r < r_c$ ) of the cluster, which are necessary in the  $M_{\text{gas}}$  and  $M_{\text{tot}}$  reconstruction process. The formula for the mass of hot ICG enclosed within a radius  $r$  is the following [15]:

$$M_{\text{gas}}(< r) = 4\pi \int \rho_{\text{gas}}(r) r^2 dr. \quad (3)$$

Since ICG is almost fully ionized (what its temperature suggests) and we expect that its structure represents the primordial composition of the Universe (*i.e.* it should be composed mainly of hydrogen and helium with small fraction of heavier elements), we are justified in the application of gas density as  $\rho_{\text{gas}} = \mu_e m_p n_e(r)$ , where  $m_p$  and  $\mu_e$  are respectively: the proton mass

and the mean molecular weight of electrons. With the assumption that the ICM gas is a perfect gas ( $P_{\text{gas}} = k_{\text{B}} T_{\text{gas}} n_{\text{gas}}$ ) composed of electrons and protons ( $n_{\text{gas}} = n_e + n_p \sim 1.83 n_e$ , where  $n_p$  is the proton number density) in a hydrostatic equilibrium with the Navarro–Frenk–White potential [16] describing matter distribution within dark matter halos, we can infer the total mass of the cluster as

$$M_{\text{tot}}(< r) = -\frac{r^2}{G\rho_{\text{gas}}(r)} \frac{dP}{dr}. \quad (4)$$

$P = (\mu_e/\mu)P_e$  is the total pressure of the ICM gas and  $\mu = \rho_{\text{gas}}/(m_{\text{u}} n_{\text{gas}}) \sim 0.6$  is the mean molecular weight (in atomic mass unit) for ionized plasma [17].

### 3. Methodology and the samples used

Theoretical formula for gas mass fraction for galaxy clusters is the following:

$$f_{\text{gas}}^{\text{th}}(z; \mathbf{p}) = \frac{KA\gamma b_0(1 + \alpha_b z)}{1 + s_0(1 + \alpha_s z)} \frac{\Omega_b}{\Omega_m} \left[ \frac{D_{\text{A}}^{\text{ACDM}}(z; \mathbf{p})}{D_{\text{A}}(z; \mathbf{p})} \right]^{1.5}. \quad (5)$$

The seven parameters in the above:  $\tilde{p} = \{K, A, \gamma, s_0, b_0, \alpha_s, \alpha_b\}$  depend on the details adopted in the  $f_{\text{gas}}$  modelling procedure [8, 18] and  $D_{\text{A}}(z; \mathbf{p}) := \frac{1}{1+z} \frac{c}{H_0} \int_0^z \frac{dz'}{E(z'; \mathbf{p})}$  is the angular diameter distance to the cluster which is the function of the cosmological parameters denoted collectively here as  $\mathbf{p}$ .  $\Lambda$ CDM model is treated as the reference for which  $E_{\Lambda\text{CDM}}^2(z; \mathbf{p}) = \Omega_m(1+z)^3 + (1 - \Omega_m)$  with standard values of the Hubble constant  $H_0 = 70 \frac{\text{km}}{\text{s} \cdot \text{Mpc}}$  and matter density parameter  $\Omega_m = 0.3$ . In our analysis, we took into account two most popular dark energy scenarios, in the assumption of spatial flatness of the Universe [19]. These are:  $w$ CDM model in which the dark energy equation of state<sup>1</sup> coefficient  $w$  is constant in time ( $E_{w\text{CDM}}^2(z; \mathbf{p}) = \Omega_m(1+z)^3 + (1 - \Omega_m)(1+z)^{3(1+w)}$ ) and the model where the  $w$  parameter evolves in time according to the Chevalier–Polarski–Linder (CPL) parametrization  $w(z) = w_0 + w_a \frac{z}{1+z}$ , which is actually a Taylor expansion with respect to the scale factor — a true degree of freedom in the Friedman–Robertson–Walker geometry [20] ( $E_{\text{CPL}}^2(z; \mathbf{p}) = \Omega_m(1+z)^3 + (1 - \Omega_m)(1+z)^{3(1+w_0+w_a)} \exp(\frac{-3w_a z}{1+z})$ ).

Cosmological parameters  $\mathbf{p}$  within each of the adopted scenarios have been adjusted in such a way that any trend in the observed value of the gas mass fraction  $f_{\text{gas}}^{\text{obs}}$  (*i.e.* its apparent evolution) with redshift vanishes [8].

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<sup>1</sup> Assuming barotropic equation of state for dark energy  $p = w\rho$ .

Best-fit values of cosmological parameters has been obtained by performing Monte Carlo simulations<sup>2</sup> looking for a minimisation of the  $\chi^2$  function<sup>3</sup>

$$\chi^2 = \sum_{i=1}^n \frac{(f_{\text{gas}}^{\text{obs}}(z_i) - f_{\text{gas}}^{\text{th}}(z, \tilde{p}, \mathbf{p}))^2}{\sigma_i^2} \quad (6)$$

with respect to  $\mathbf{p} = \{\Omega_m, w\}$  in  $w$ CDM model and  $\mathbf{p} = \{\Omega_m, w_0, w_a\}$  in CPL model, and marginalized over nuisance parameters  $\tilde{p}$ . In our analysis, we used  $\Omega_b = 0.04931$  from [19]. The observed values of the gas mass fraction  $f_{\text{gas}}^{\text{obs}}$  was taken from two independent measurements of hot and dynamically relaxed galaxy cluster samples from Chandra X-ray observations: the first one was performed by Allen *et al.* [8] on the basis of 42 clusters with redshifts spanning the range of  $0.05 < z < 1.1$ , and the second one — by Landry *et al.* [21] for 35 clusters selected from the Brightest Cluster Sample and the Extended Brightest Cluster Sample at redshifts  $0.15 < z < 0.30$ . Both  $f_{\text{gas}}^{\text{obs}}$  measurements have been performed with similar assumptions and methods what gave us an opportunity to perform a joint analysis which, from statistical point of view, should ensure better fitting procedure of cosmological parameters.

One may notice that galaxy clusters from selected samples cover rather low redshift range which seems to work against their choice for cosmology. However, in the light of the cosmographic approach, when the evolution of deceleration parameter  $q(z)$  with redshift is taken into account, galaxy clusters are in fact promising. In particular, one may find works suggesting that the recent accelerated expansion of the Universe may be transient, what is observed as changes in the sign of  $q(z)$  for redshifts up to  $z \sim 0.5$  [22, 23]. Such behaviour of  $q(z)$  revealing a possible present slowing down of the cosmic expansion is explained as an artifact of an improperly selected sample [24], but this problem remains open. Thus, one may see that the famous opinion of Sandage [25] that the modern observational cosmology is actually a quest for only two numbers (*i.e.* Hubble  $H_0$  and deceleration  $q_0$  parameters at present epoch) is still actual. These numbers are widely known as the main cosmographic parameters — their knowledge allows to determine time dependence of the scale factor  $a(t)$  and thus, dynamical behaviour of the Universe, clearly seen in the traditionally written Taylor expansion in the vicinity of the present time  $t_0$

$$a(t) = 1 + H_0(t - t_0) - \frac{1}{2}q_0H_0^2(t - t_0)^2 + \dots, \quad (7)$$

where  $H_0 = \dot{a}(t_0)/a(t_0)$  is the Hubble parameter and the first non-linear correction  $q_0 = -\ddot{a}(t_0)a(t_0)/\dot{a}^2(t_0)$  is known as deceleration parameter.

<sup>2</sup> Written in R.

<sup>3</sup>  $n$  is a size of the sample used.

To investigate recent (*i.e.* at low redshift) expansion history of the Universe, one has to allow for the possible evolution of the deceleration parameter with time

$$q(t) = -\frac{1}{H(t)} \frac{\ddot{a}(t)}{\dot{a}(t)} = -1 - \frac{1}{H(t)} \left( H(t) + \frac{1}{H(t)} \frac{dH(t)}{dt} \right). \quad (8)$$

In cosmology, redshift  $z$  is the true observable thus it is necessary from observational point of view to translate the formula for  $q(t)$  into the function of  $z$

$$q(z; \mathbf{p}) = -1 + \frac{1+z}{E(z; \mathbf{p})} \frac{dE(z; \mathbf{p})}{dz}. \quad (9)$$

Here, we used the known relation between redshift  $z$  and the scale factor  $a(t)$  (*i.e.*  $1+z = 1/a(t)$ ) to obtain another one:  $dt = -1/H(z)(1+z)dz$ . The Hubble function is  $H(z) = H_0 E(z; \mathbf{p})$  with cosmological model parameters denoted as in the above by  $\mathbf{p}$ . This justifies the formula for the deceleration parameter in the last equation to be written explicitly as a function of  $\mathbf{p}$ .

#### 4. Results and conclusions

This work is an extension of our previous analysis carried out on a smaller sample of galaxy clusters to obtain values only for two cosmological parameters:  $\Omega_m$  and  $\Omega_b$  [26]. The best-fit values of cosmological parameters for the galaxy clusters  $f_{\text{gas}}$  data from [8] and [21], obtained by us through the analysis detailed above, are the following:  $\Omega_m = 0.3695^{+0.1121}_{-0.173}$ ,  $H_0 = 66.74^{+28.48}_{-42.15}$ ,  $w = -0.78^{+0.1695}_{-0.2615}$  ( $\chi^2/\text{d.o.f.} = 1.7465$ ) in the  $w$ CDM scenario and  $\Omega_m = 0.2523^{+0.1738}_{-0.0694}$ ,  $H_0 = 67.29^{+24.27}_{-29.98}$ ,  $w_0 = -0.86^{+0.4541}_{-0.3245}$ ,  $w_a = 0.6948^{+0.5226}_{-0.892}$  ( $\chi^2/\text{d.o.f.} = 1.7456$ ) for time-varying equation of state within CPL scenario. Comparing these results with the latest ones from the Planck data based on the analysis of CMB anisotropies [19], supernovae of type Ia data [3] or strong gravitational lensing data [27], one can find strong agreement between them. This, in turn, speaks in favour of the use of galaxy clusters for cosmology as a promising tool, complementary to other well-established and highly-confident methods such as supernovae of type Ia, CMB or barion acoustic oscillations. Further works in this area are in progress. In particular, the quality of  $f_{\text{gas}}$  measurements has been significantly improved with the use of the thermal Sunyaev–Zel’dovich (SZ) effect [28] in a complementary way to the X-ray data [29]. The presence of high-energy electrons associated with ICG disturbs the energy distribution of low-energy cosmic microwave background (CMB) photons via the inverse Compton scattering. This can be seen as the apparent change in CMB temperature distribution  $T_{\text{CMB}}$  [29] which, in turn, can be translated

into the temperature and pressure profiles of ICG. Such a distortion of CMB blackbody spectrum, even if it is rather small (less than about 1 mK) but is very specific which implies the fact that SZ effect can be measurable to a high level of precision. Since directly from its nature the SZ effect does not depend on the redshift of a cluster, the temperature and pressure profiles of ICG obtained on the basis of SZ data have relatively low scatter which should allow us to obtain better estimates of the cosmological parameters. This will be the subject of our next paper which is now in a preparation process.

Results of our analysis concerning the evolution of deceleration parameter with redshift  $q(z)$  on the basis of estimated best-fit cosmological parameters for  $w$ CDM and CPL models are shown in Fig. 1 and Fig. 2, respectively. In comparison to the  $q(z)$  curve in the Fig. 4 in [23] reconstructed with the Union 2.1 sample of type Ia supernovae, one may find a similar trend revealing no evidence for a low-redshift transition of the deceleration parameter and suggesting that its possible evolution toward decelerating phase of Universe in the near future is rather unlikely. This will also be the subject of our further investigations.

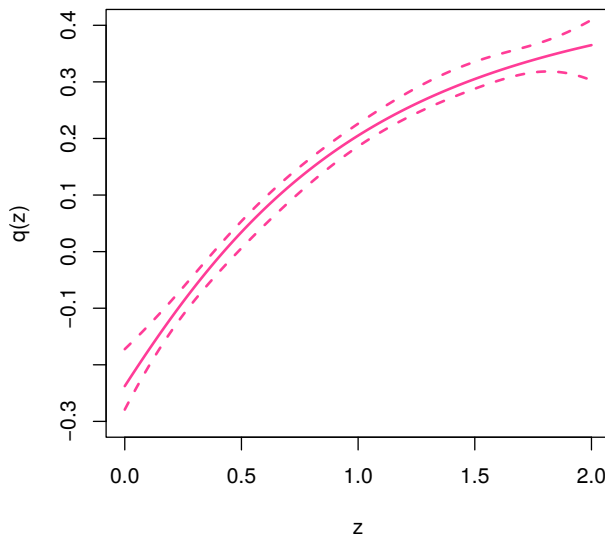


Fig. 1. Deceleration parameter as a function of redshift for best-fit values of cosmological parameters (solid line) obtained on the basis of joint analysis of  $f_{\text{gas}}$  data from [8] and [21] for  $w$ CDM cosmology. Dashed line shows error propagation at  $2\sigma$  around estimated values of cosmological parameters.



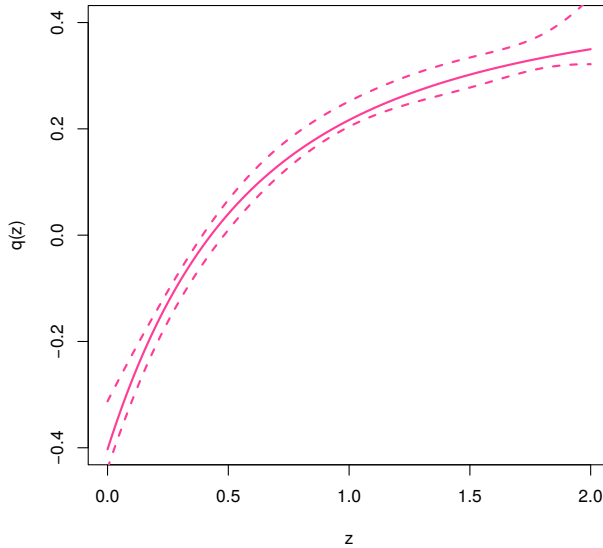


Fig. 2. Deceleration parameter as a function of redshift for best-fit values of cosmological parameters (solid line) obtained on the basis of joint analysis of  $f_{\text{gas}}$  data from [8] and [21] for CPL scenario. Dashed line shows error propagation at  $2\sigma$  around estimated values of cosmological parameters.

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